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# Improving excitation and inversion accuracy by optimized RF pulse using genetic algorithm

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**Abstract**

In this study, a Genetic Algorithm (GA) is introduced to optimize the multidimensional spatial selective RF pulse to reduce the passband and stopband errors of excitation profile while limiting the transition width. This method is also used to diminish the nonlinearity effect of the Bloch equation for large tip angle excitation pulse design. The RF pulse is first designed by the *k*-space method and then coded into float strings to form an initial population. GA operators are then applied to this population to perform evolution, which is an optimization process. In this process, an evaluation function defined as the sum of the reciprocal of passband and stopband errors is used to assess the fitness value of each individual, so as to find the best individual in current generation. It is possible to optimize the RF pulse after a number of iterations. Simulation results of the Bloch equation show that in a 90° excitation pulse design, compared with the *k*-space method, a GA-optimized RF pulse can reduce the passband and stopband error by 12% and 3%, respectively, while maintaining the transition width within 2 cm (about 12% of the whole 32 cm FOV). In a 180° inversion pulse design, the passband error can be reduced by 43%, while the transition is also kept at 2 cm in a whole 32 cm FOV.


*Keywords:* Genetic algorithm; Multidimensional spatial selective pulse; Passband; Stopband

## 1. Introduction

Multidimensional spatially selective RF pulses have been widely used in MRI. They are able to limit the electromagnetic signal emitted from the imaging object within arbitrarily shaped and spatially restricted areas, and are commonly used for excitation [1–4], refocusing and inversion [5–7]. They can be designed using the *k*-space method [1,2]. However, the finite length RF pulse may cause ripples in the excitation profile, and the undesired signals from the transition may degrade the image quality. Also, since it is based on Fourier Transform, the nonlinearity of the Bloch equation has a considerable impact on the excitation profile when it is used for large tip angle excitation.

The Shinnar Le Roux (SLR) method has been widely used in slice selection [8], allowing the designer to make trade-offs among parameters such as passband error, stopband error and transition width. This method has also been extended to the design of multi-dimensional excitation pulses [9,10], but is limited in uniform sampling *k*-space trajectories.

Between 1987 and 1991 several studies [11–13] have applied a Genetic Algorithm (GA) to design new types of RF pulse. This pulse is composed of different shaped pulses, which could be chosen and adjusted by a GA. The shaped pulses are mainly Gaussian and Sinc function, which constrain the whole pulse shape.

In recent years, GAs have been widely used in filter design [14–17] to adjust the filter amplitudes directly and to optimize the Finite Impulse Response (FIR) filter to fit ideal frequency response. The RF pulse design is very similar to the FIR filter design, in which the passband and stopband correspond to the in-slice and out-slice, respectively. Here, the finite impulse response corresponds to the amplitude of the RF pulse. We could firstly design an RF pulse using the *k*-space method, and then carefully

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adjust the amplitude of the RF pulses within a small range, to reduce the ripples induced by truncated Fourier coefficients, and diminish the effect caused by the Bloch equation nonlinearity effect. In this study, a novel GA optimization approach, which adjusts the RF pulse directly, is proposed to optimize the multidimensional spatial selective RF pulse designed by the $k$-space method to reduce the passband and stopband errors in the excitation profile and to limit the width of the transition band. This method allows trade-offs among the different parameters, and can be applied to arbitrary $k$-space trajectory design. The method can also be used to optimize the excitation pulses with a large tip angle as it is able to diminish the Bloch equation nonlinearity effect.

## 2. Methods and procedure

There are two main steps in our method: (a) design of the gradient waveforms and the RF pulse using the $k$-space method; and (b) adjustment of the RF pulse amplitude within a certain range using a GA to improve the excitation profile. Here, the RF pulse is a series of discrete values (e.g. 512 discrete points).

In $k$-space method [1,2], a Fourier Transform approximation relation between the desired excitation profile and the applied RF pulse is established. The RF pulse determines the weighting function, while gradient waveforms define the $k$-space trajectory. Thus, the desired profile can be approximated by Fourier Transform of spatial frequency weighting function multiplied by a sampling function. In this work, spiral trajectory is adopted, and defined as:

$$k(t) = k_{\max}(1-t)\mathrm{e}^{i2\pi N(1-t)} \quad (1)$$

where $k_{\max}$ and $N$ are the max value of $k$-space and spiral trajectory turns. To generate a cylinder profile, the weighting function is defined as following [4]:

$$W(k(t)) = J_1(k(t))/k(t) \quad (2)$$

where $J_1$ is the first order Bessel function of first kind. Thus, the RF pulse is:

$$B_1(t) = -\mathrm{i}|g(t)|W(k(t)) \quad (3)$$

where $g(t) = (1/\gamma)\mathrm{d}k(t)/\mathrm{d}t$.

GA is an optimization method which imitates evolutionary process with genetic operators: reproduction, crossover and mutation. First, the solutions are coded as an 'individual' binary or float string of a certain length. In this study, a float string is described as:

$$v = [p_1, p_2, \cdots, p_n] \quad (4)$$

where $v$ denotes the individual, $p_i$ is a float number denoting the amplitude of the discrete values of RF pulse, and $n$ is the individual length denoting the number of points. In $k$-space design, the RF pulse is composed of hundreds of discrete points. If all of them are adjusted by the GA, the calculation will be prohibitively time-consuming. Therefore, only $n$ points at uniform intervals are selected for adjustment, while the remainder are interpolated using piecewise cubic spline interpolation.

This involves two problems, viz. the setting of the individual length $n$ and the pulse amplitudes adjustment range $\delta$. In GA, to cover the solution space as far as possible, a large number $n$ should be adopted, which will leads to a large initial population size. Because GA evolution duration is proportional to the population size, a prohibitive amount of time for optimization will be required if $n$ has too large a value. The value of $n$ must therefore be kept within certain limits. However, piecewise cubic spline interpolation has been chosen to fill in the data between two adjusted points. This requires estimation of all the amplitudes except the $n$ point values, which are accurately determined by the GA. If too small a value for $n$ is used, large estimation errors may be produced. Therefore, it is necessary to select an appropriate value for $n$ by trading off between these two considerations.

The adjustment range $\delta$ is set within $\pm 10\%$ of the original amplitude of the point. Because Fourier coefficients in the center area of the frequency domain, which correspond to the discrete points of the RF pulse, are much larger than those of truncated area, the adjustment value should be within a small range.

Second, an evaluation function is defined to assess the fitness of each individual. Normally, the higher the fitness value, the better the individual. To minimize the errors in passband and stopband and simultaneously limit the transition band width, the evaluation function is defined as:

$$f(v) = a/E_p(v) + b/E_s(v) \quad (5)$$

where $E_p(v)$ and $E_s(v)$ are the passband error and stopband error of individual $v$, respectively; $a$ and $b$ denote constants in the closed interval [1,10], which can be set at the appropriate value to trade-off between the actual requirements of passband and stopband. For 90° excitation, the error $E_p(v)$ and $E_s(v)$ are defined as following:

$$E_p(v) = \max_{D_p}(|M_{\perp}(v,x)| - 1) \quad (6)$$

$$E_s(v) = \max_{D_s}(|M_{\perp}(v,x)| - 0) \quad (7)$$

where $D_p$ and $D_s$ are the desired passband and stopband area, respectively. The width of the transition band can be limited as a proper value by setting the area of $D_p$ and $D_s$ before optimization. $M_{\perp}(v,x)$ is the transverse magnetization after excitation [1]:

$$M_{\perp}(v,x) = i\gamma M_0 \int_0^{\mathrm{T}} B_1(v,t)\mathrm{e}^{-\mathrm{i}\gamma x\cdot\int_t^{\mathrm{T}} G(s)\mathrm{d}s}\mathrm{d}t \quad (8)$$

where $\gamma$ is the gyromagnetic ratio, $M_0$ denotes the equilibrium longitudinal magnetization, $B_1(v, t)$ and $G(s)$ are the RF pulse envelope function of the individual $v$ and the gradient waveform, respectively. For simplicity, the equilibrium magnetization $M_0$ is assumed to be 1.

For 180° inversion, the error $E_p(v)$ and $E_s(v)$ are defined as:

$$E_p(v) = \max_{D_p}(M_z(v,x) - (-1)) \tag{9}$$

$$E_s(v) = \max_{D_s}(M_z(v,x) - 1) \tag{10}$$

Usually, there are two definitions of error: max and average. The max value reflects the worst point in the excitation profile. Given this, it is certain that the profile at other positions is better. The average value reflects the total performance in the excitation profile, but the largest error is unknown. Therefore, max is chosen to evaluate the excitation profile error.

Third, three proper genetic operators and two relative parameters are chosen for evolution, these are described below:

(1) The roulette wheel selection method is adopted as the reproduction operator, which is used for selecting elitist individuals into a mating pool to perform crossover and mutation. Assuming there are $M$ individuals $v_1, v_2, \ldots, v_M$ in a population, the fitness value of each individual $v_i$ is denoted by $f(v_i)$. The selection probability of an individual is proportional to its fitness value:

$$P\left\{T_s(\vec{v}) = v_i\right\} = \frac{f(v_i)}{\sum_{j=1^M f(v_j)}} \tag{11}$$

where $T_s$ and $\vec{v}$ denote the reproduction operator and population ($v_1$,$v_2$, $\cdots$ ,$v_M$), respectively. Eq. (11) indicates that the higher the fitness value, the larger the probability is for an individual being selected to perform evolution. This follows the evolution principle 'Survival of the Fittest'.

(2) Heuristic crossover [18] takes two parents and performs an extrapolation in the direction of the better parent:

$$v_{\text{new}} = r \cdot (v_i - v_j) + v_i \tag{12}$$

where $r$ is a random number between 0 and 1, and $f(v_i) \geqslant f(v_j)$. It is possible for heuristic crossover to generate a $v_{\text{new}}$ out of the range. If no new solution is found after three attempts, no offspring will be generated. The advantage of heuristic crossover is it can search a solution in a more promising direction towards the optimal solution.

(3) Non-uniform mutation [18] is adopted, and is depicted in the following:

$$p'_k = \begin{cases} p_k + \Delta(t, UB - p_k) & r_D = 0 \\ p_k - \Delta(t, p_k - LB) & r_D = 1 \end{cases} \tag{13}$$

where LB and UB are the lower and upper bounds of variable $p_k$ which is defined in Eq. (4); $r_D$ and $t$ denote a random digit and the present generation number, respectively, $p'_k$ is the mutation result. The function $\Delta$ is a non-uniform probability distribution function, which narrows to a point distribution as the generation number approaches the maximum generation:

$$\Delta(t,s) = s \cdot r \cdot \left(1 - \frac{t}{\mathrm{T}}\right)^b \tag{14}$$

where $r$ is random real number between 0 and 1, $b$ is a parameter determining the degree of dependency on iteration number, we used $b = 3$ here. T is the max generation number.

(4) Before optimization, it is a fundamental decision for choosing population size $M$. There is no general theory for setting optimal population size. Usually, it is determined by experiments or experience. In this work, 10 different population size setting (from 10 to 100 at interval 10) were tested, and finally we found that the GA with population size of 30 reaches the stop criterion faster than other GAs.

(5) Stop criterion indicates when the evolution stops. Because in our research we care more about the passband error, the stop criterion was set at passband error $E_p(v)$ equaling a specified value $E_0$. For different application requirement $E_0$ will be different. In this work, $E_0$ was 0.02 for excitation pulse optimization and 0.04 for inversion pulse.

Following the above steps, the RF pulse can be optimized by genetic evolution iteratively. At the beginning, an initial population with $M$ individuals is generated randomly, and the fitness value of each individual is evaluated. The individuals with higher fitness weigh much more than lower fitness individuals. Therefore, according to the roulette selection rule, the GA reproduces the high quality individuals to the mating pool, although the reproduction does not improve the fitness of individuals. Then, these high quality individuals are crossed so that the distribution of schema is modified, which make it possible to generate some higher fitness individuals. Each RF pulse is represented by an individual composed of genes, therefore the RF pulse can be regarded as composed of many discrete points. In the crossover operation, the amplitudes of these discrete points in two RF pulses are changed to generate new RF pulses. This makes it possible to attain some better RF pulses. According to Holland's Schema Theorem [19], the best individual will be finally found by the genetic evolution. Next, mutation is applied to explore larger searching space. The RF pulse can also be regarded as many discrete points. The mutation operator changing genes of an individual, makes it possible to find some higher performance RF pulses. According to evolution principle, these three genetic operators—reproduction, then crossover, and finally mutation—should be in sequence.

In addition, in order to insure the fitness of the best individual in each generation monotonically increases, the best individual in each generation will be reproduced to the next generation without evolution. Finally, after evolution, the best individual in the final generation will be achieved. This best individual is the optimized RF pulse. The algorithm flowchart is shown in Fig. 1.

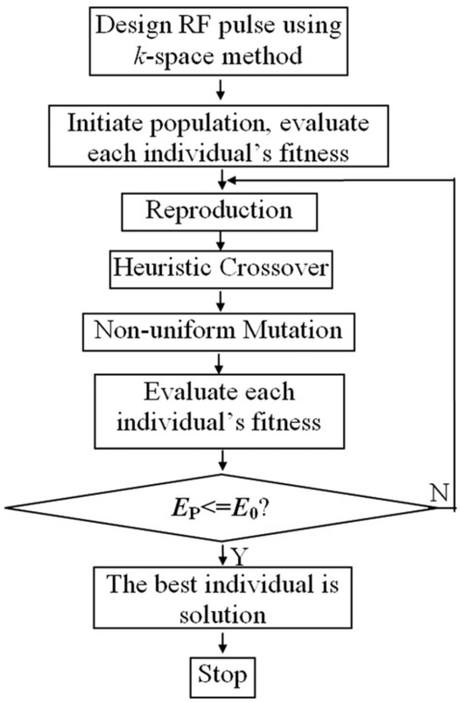


Fig. 1. Flowchart of GA optimized RF pulse design.

## 3. Simulation

This GA evolution method can be used to optimize different types of RF pulse. In this study, a 2D selective pulse using spiral trajectory was designed by the *k*-space method and optimized by GA. Then a numerical simulation of the Bloch equation was performed to examine the accuracy of the actual excitation profiles. Finally, the excitation profile errors of both the *k*-space method designed RF pulse and the GA-optimized RF pulse are calculated and compared to verify the effectiveness of the proposed method.

In the 2D selective pulse design, two pulses were designed, one for 90° flip angle excitation and one for 180° flip angle. Both pulses were based on the same spiral *k*-space trajectory with 12 turns, and the max value of *k*-space was 0.5 cycle/cm. The following parameters were the same for both pulses. The desired excitation profile (Figs. 3b and 4b) was a 7 cm diameter cylinder with FOV of 32 cm. The transition band width was set as 2 cm with the magnetization dropping down from 0.97 to 0.02 for 90° pulse and rising from −0.98 to 0.98 for 180° pulse. A gradient pulse with 4G/cm amplitude and 15G/cm/ms slew rate was used. With the GA, after several tests, the parameters were set as: the population size $M$ was 30, the individual length $n$ was 10 and the terminal condition was the passband error equaling 0.02 for 90° pulse and the passband error equaling 0.04 for 180° pulse. The coefficient $a$ and $b$ in Eq. (5) were set as 1 and 10 for 90° pulse, 4 and 1 for 180° pulse.

In addition, because a GA is stochastic, the same parameters used on the same problem by the same GA will generally yield different results. If the final result of each evolution varies within too great a range, the GA may be unstable and virtually unusable in the application. It is important to clearly indicate the range of the result after GA optimization. Here, we used the mean and standard deviation analysis to evaluate the convergence stability of the GA. Simulations with the same parameters were made several times, and the excitation profile errors of the best population after each evolution were recorded. This set of data was then used to calculate the mean and standard deviation of the optimization results. The mean and standard deviation indicate the expectation and varying range of the data set, respectively. The smaller the standard deviation, the greater the stability of the performance of this method. As it is necessary to use as large a data set as possible to achieve robust statistical results, we made 200 simulations in this study to support our statistical analysis.

## 4. Results

The genetic evolution process of the 90° pulse and 180° pulse are shown in Fig. 2, which illustrates the improve-

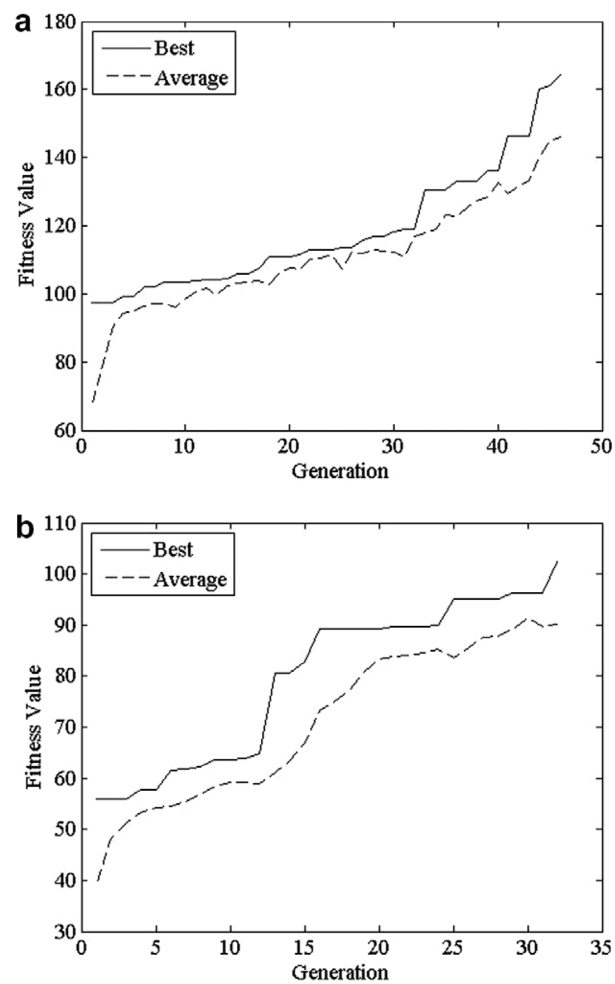


Fig. 2. (a) Evolution process of best population fitness (solid line) and average fitness (dash line) in each generation, for 90° pulse. (b) Evolution process of best population fitness (solid line) and average fitness (dash line) in each generation, for 180° pulse.

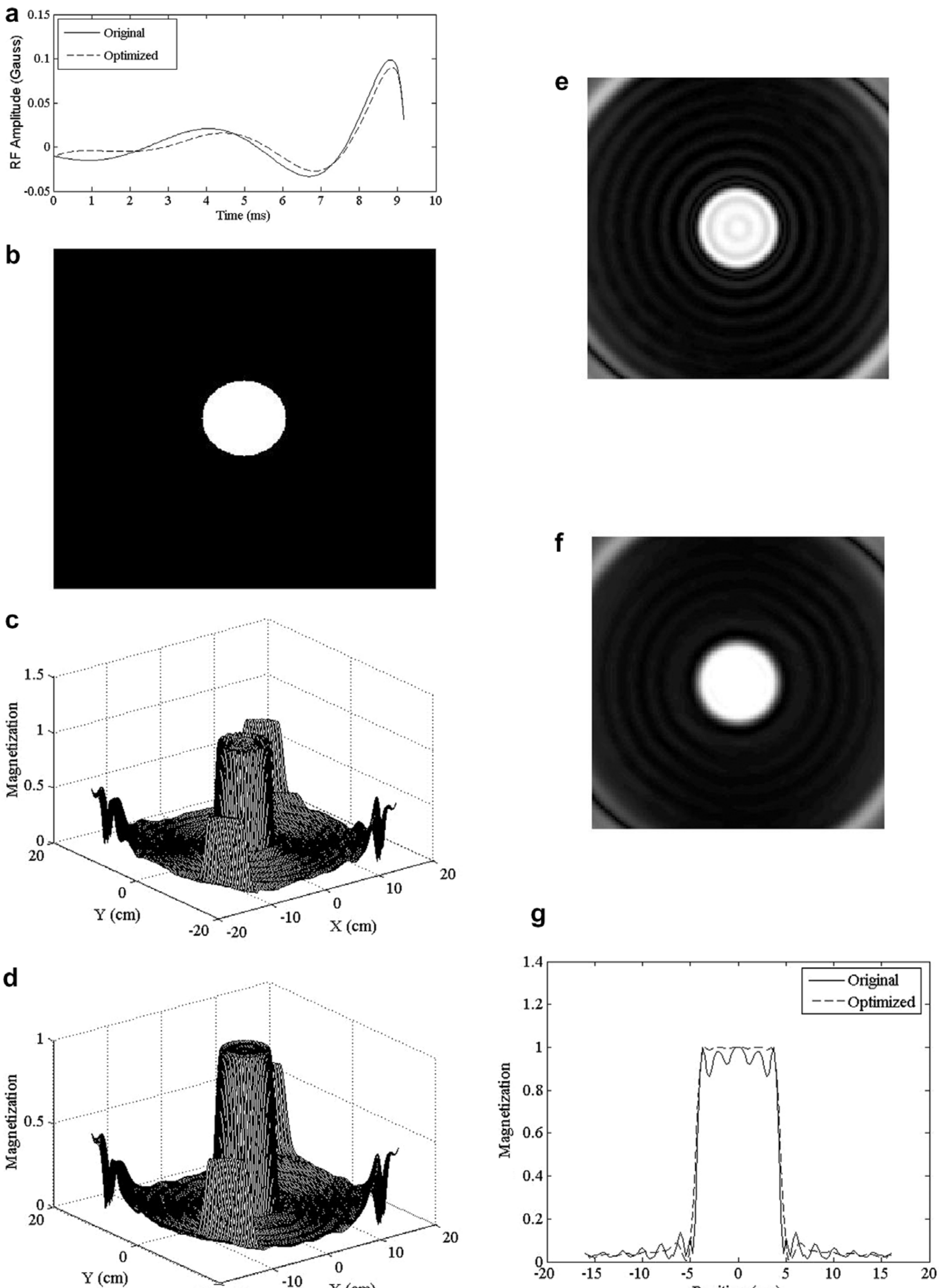


Fig. 3. Simulation comparison of the transverse magnetization produced by 90° RF pulses. (a) *k*-Space method designed RF pulse (solid line); GA optimized RF pulse (dash line); (b) desired 2D excitation profile; (c) excited volume of *k*-space method designed RF pulse; (d) excited volume of GA optimized RF pulse; (e) excitation profile of *k*-space method designed RF pulse; (f) excitation profile of GA optimized RF pulse; (g) 1D excitation profile of original RF pulse (solid line) and optimized RF pulse (dash line).

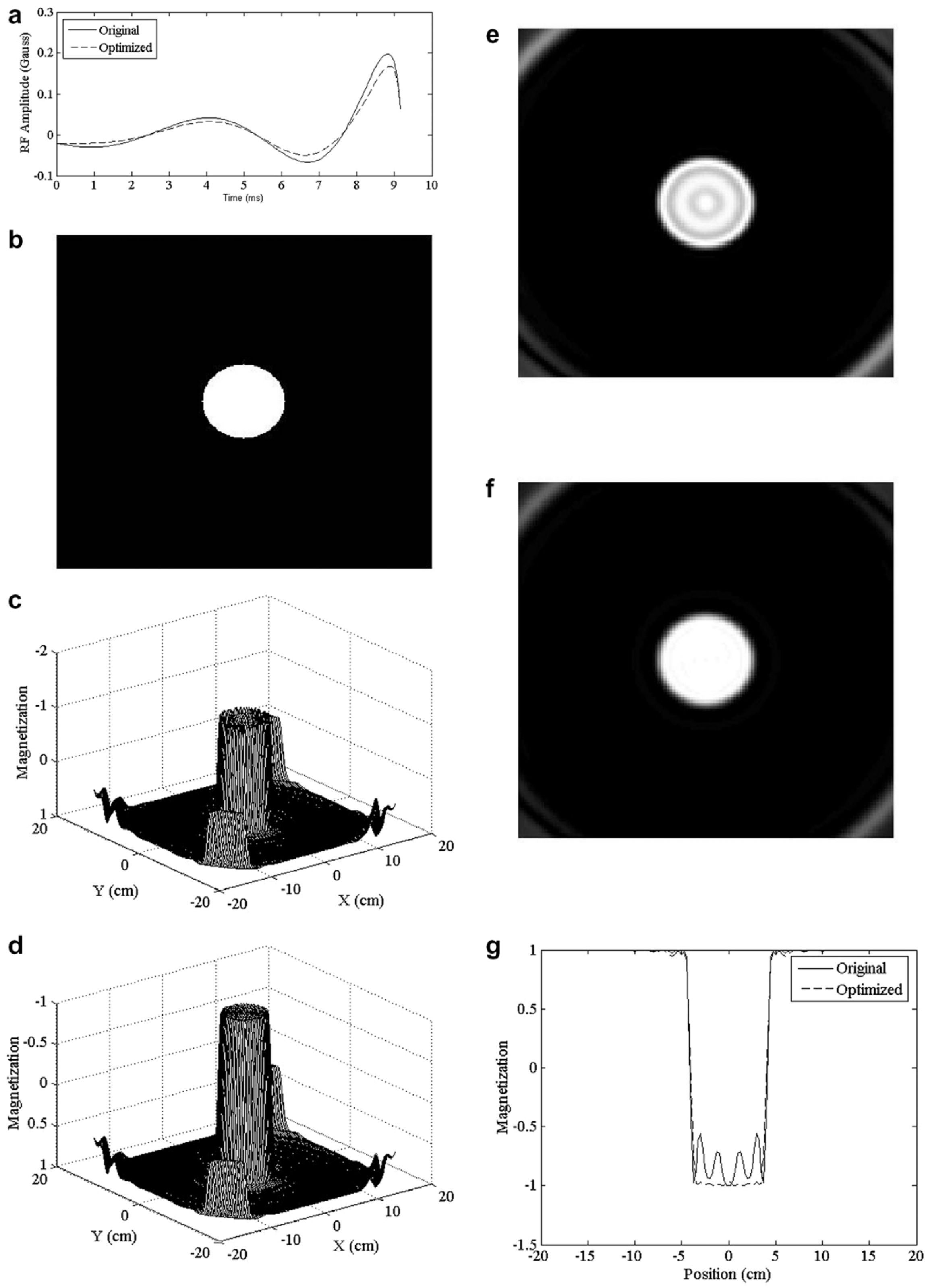


Fig. 4. Simulation comparison of the longitudinal magnetization produced by 180° RF pulses (a) *k*-Space method designed RF pulse (solid line); GA optimized RF pulse (dash line); (b) desired 2D inversion profile; (c) excited volume of *k*-space method designed RF pulse; (d) excited volume of GA optimized RF pulse; (e) inversion profile of *k*-space method designed RF pulse; (f) inversion profile of GA optimized RF pulse; (g) 1D inversion profile of original RF pulse (solid line) and optimized RF pulse (dash line).

ment in best individual fitness and average fitness at each generation. The best individual fitness (solid curve) of each generation remained the best for a period and was then replaced by a better one, corresponding to one jump in the best fitness curve. Prior to each jump, the average fitness value (dash curve) increased because the GA operators were adjusting the value of the worse individuals to approach that of the best individual. After each jump, the average fitness was always small, because the jump of the best individual decreased population diversity. When a GA is used, best individual fitness is much more important than average fitness, because the optimization goal is to approach the maximum. Fig. 2 shows that the fitness value of the best individual increased monotonically. Fig. 2a shows the evolution of the excitation pulse, the GA reached the stop criterion after 46 generations, taking about 6 min; Fig. 2b shows the inversion pulse evolution, the GA reached the stop criterion after 32 generations, taking about 5 min.

Fig. 3 shows the results of the numerical simulation of the transverse magnetization produced by the original and the optimized 90° RF pulses. Fig. 3a shows the 2D selective RF pulses designed by the *k*-space method (solid line) and optimized by the GA (dash line). It is noteworthy that the optimized RF pulse was slightly different from the original pulse, which affected the actual excitation profile. Fig. 4c and d show the excited volumes generated by the two pulses. To make the comparison clearer, the 2D excitation profiles are shown in Fig. 3e and f. It is evident that the excitation accuracy was greatly improved using the optimized 90° RF pulses. According to Eqs. (6) and (7), the passband error fell from 0.14 (before optimization) to 0.02 (after optimization) while the stopband error fell from 0.14 (before optimization) to 0.11 (after optimization) within 6 min. The transition was limited within 2 cm when the magnetization dropped from 0.98 to 0.02. The corresponding 1D excitation profiles of the central line of the slice are plotted in Fig. 3g, which shows the passband, stopband and transition more clearly.

According to the statistical analysis of the 200 simulations of the 90° RF pulses optimization using the same parameters, the mean and standard deviation of the generation number for GA reaching stop criterion were 42 and 3.24, respectively, while those of the stopband error were 0.12 and 0.02. The standard deviations for stop generation number and stopband error were only 7.7% and 16.7% of the means, indicating that this algorithm is stable enough for convergence in practical 90° pulse design.

The numerical simulation of longitudinal magnetization of 180° RF pulses is shown in Fig. 4. Fig. 4a shows the 2D selective RF pulses designed by the *k*-space method (solid line) and optimized by the GA (dash line). The difference between the original pulse and the optimized pulse affected the actual excited volume, which is shown in Fig. 4c and d. To make the comparison clearer, 2D excitation profiles are shown in Fig. 4e and f. Again, it is clear that the excitation accuracy was greatly improved using the optimized 180° RF pulses. According to Eqs. (9) and (10), the passband error fell from 0.47 (before optimization) to 0.04 (after optimization) while the stopband error rose from 0.04 (before optimization) to 0.06 (after optimization) within 5 min. The transition was limited within 2 cm as the magnetization rose from −0.98 to 0.98. The corresponding 1D excitation profiles of the central line of the slice are plotted in Fig. 4g, which shows the passband, stopband and transition more clearly. Although the ripples in the stopband of the optimized pulse were slightly larger than those of the original pulse, the passband profile shows a considerable improvement.

According to the results of the statistical analysis for the 180° pulse optimization, the mean and standard deviation of the stop generation number were 36 and 3.7, respectively, while the stopband errors were 0.06 and 0.003, respectively. The standard deviations for the stop generation number and the stopband error were only 10% and 5% of the means. This indicates that this algorithm is stable enough for convergence for practical inversion pulse design.

## 5. Discussion and conclusion

In this study, a GA has been used to optimize RF pulse designed by *k*-space method to reduce the passband and stopband errors of excitation profile while limiting the width of the transition band. The simulation results of a 90° excitation pulse have shown that optimizing the GA can reduce the errors of the excitation profile significantly while keeping the transition width within the desired values. In the simulation of a 180° inversion pulse, the longitudinal inversion profile of the original pulse was degraded due to the nonlinearity of the Bloch Equation. After optimization, although there were some small ripples in the stopband, the nonlinearity effect was reduced and the accuracy of the profile was greatly improved. Although this method has been only tested on a 2D selective pulse using spiral trajectory, it could also be easily applied to a slice selection pulse and a 3D selective pulse and used for an arbitrary *k*-space trajectory.

This method allows us to trade-off between passband error, stopband error and transition width. If the parameters are set too small, the optimization process will require far more iterations, and it might even become impossible for the GA to find the proper solution. Because fixing the pulse length also fixes the upper bound of excitation profile accuracy, roughly the same as in the FIR filter design, shortening the transition width and reducing the error would require a longer filter length. Theoretically, an adaptive GA approach which optimizes both the RF pulse length and its amplitude might produce a design which could search for better solutions. But it would probably require a considerably larger searching space and much longer evolution time.

During the GA optimization process, the adjusted RF pulse may induce larger RF power than the original RF pulse, which could lead to more power deposition in the imaging object. Therefore, SAR should be kept to a tolerable value when reducing the excitation profile errors. One possible solution is to introduce the SAR estimation into the definition of the evaluation function. However, this would require trade-offs between more parameters (SAR, passband, stopband and transition), and this would be a much more involved and complex process.

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